Stability Analysis for Nonlinear PDEs and Multiscale Applications.

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Book of Abstracts

Dallas Albritton, University of Wisconsin-Madison

Progress Report on Instability and Non-uniqueness in Incompressible Fluids

Over the past decade, mathematical fluid dynamics has seen remarkable progress in an unexpected direction: non-uniqueness of solutions to the fundamental PDEs of incompressible fluids, namely, the Euler and Navier-Stokes equations. I will explain the state-of-the-art in this direction, with a particular focus on the relationship between instability and non-uniqueness, including our proof with E. Brue and M. Colombo that Leray-Hopf solutions of the forced Navier-Stokes equations are not unique. Time permitting, I will present forthcoming work with Colombo and Mescolini in which we investigate the non-existence of a selection principle for the forced 2D Euler equations in the vanishing viscosity limit

Myoungjean Bae, Korea Advanced Institute of Science and Technology (KAIST)

Multi-dimensional solutions to steady Euler-Poisson system (Part 2)

Firstly, I introduce a boundary value problem of a linear equation with a degeneracy of Keldysh type, and prove the existence of $H^m (m \geq 4)$ solution provided that the Kuzmin coefficient of order k is bounded above by a negative constant for all $1 \leq k \leq m$. As an application, I present a result on the existence of two dimensional regular transonic solution to steady Euler-Poisson system in the second part of my talk. Finally, I shall close my talk with an open question on types of sonic interfaces.

Gui-Qiang Chen, University of Oxford

Entropy Analysis and Singularities of Entropy Solutions for Nonlinear Partial Differential Equations

In this talk, we present some recent developments in solving several longstanding open problems involving the singularities of entropy solutions for nonlinear conservation laws and related nonlinear partial differential equations through entropy analysis and associated methods. These problems especially include the minimal entropy conditions for well-posedness, the understanding of underlying phenomena of cavitation and concentration, and the rigorous analysis for entropy solutions via the theory of divergence-measure fields, among others. Further related topics, perspectives, and open problems will also be addressed.

Jeffrey Cheng, University of Texas at Austin

L^2-stability and uniqueness for scalar conservation laws with concave-convex fluxes

We study stability properties of solutions to $1-d$ scalar conservation laws with a class of non-convex fluxes. Using the theory of a-contraction with shifts, we show L^2-stability for shocks among a class of large perturbations, and give estimates on the weight coefficient a in regimes where the shock amplitude is both large and small. Then, we use these estimates as a building block to show a uniqueness theorem under reduced entropy conditions for weak solutions to the conservation law via a modified front tracking algorithm. The proof is inspired by an analogous program carried out in the 2×2 system setting by Chen, Golding, Krupa, and Vasseur.

Cooper Faile, University of Texas at Austin

Criteria for Stability of Intermediate Shocks

I will discuss the theory of a-contraction with shifts, a method developed by Alexis Vasseur and collaborators to extend the Kruzkov/Dafermos-DiPerna stability results. To this point, most of the work of a-contraction has been done with scalar laws, systems of two equations, and the extremal families of systems. I will motivate the difficulties of extending these results to shocks of intermediate families and present necessary and sufficient conditions for a-contraction to hold (against small perturbations) for small shocks of these families.

Wilfrid Gangbo, University of California, Los Angeles

Viscosity solutions in non-commutative variables

Abstract: We consider optimal control problems where the observables are d–tuples of self–adjoint matrices (instead of $(R^d)^n$) and the symmetry group is the set of unitary matrices (instead of the set of permutations of n letters). This lead us to the study Hamilton-Jacobi equations satisfying some invariance properties in the setting of non-commutative variables. We consider a first type of randomness which comes from d–tuples of n by n random matrices filled with independent Brownian motion entries, the so-called Gaussian Unitary Ensembles. The second type of randomness is a single classical Brownian motion that is shared along the diagonal of the matrices, referred to as a common noise. We comment on the invariance properties satisfied by the limiting Hamilton–Jacobi equations. (This talk is based on works in collaboration with D. Jekel, K. Nam and A. Palmer).

Pierre-Emmanuel Jabin, Pennsylvania State University

A Duality method for mean-field limits with singular interactions

We introduce a new approach to justify mean-field limits for first- and second-order particle systems with singular interactions. It is based on a duality approach combined with the analysis of linearized dual correlations, and it allows to cover for the first time arbitrary square-integrable interaction forces allowing to consider a vanishing temperature parameter. For first-order particle systems, we also indicate how it allows to recover the mean field limit to the 2D Euler and Navier-Stokes equations. This is a joint joint with D. Bresch and M. Duerinckx.

Tian Jing, University of Michigan

Existence of strong solution to the two-phase magnetohydrodynamic equations

In this talk, we consider the two-phase magnetohydrodynamic equations. Two incompressible, viscous and conductive fluids are placed in magnetic field and the surface tension on interface is considered. The concept of strong solution and its existence results will be introduced. The proof is based on the Hanzawa transformation of the free-boundary problem and the construction of a contraction mapping. This talk is based on the joint work with Dehua Wang.

Chanwoo Kim, University of Wisconsin-Madison

Convergence from Boltzmann to 2D incompressible Euler

Abstract: Without any expansions (e.g., Hilbert expansion), we prove the convergence of vorticity from the Boltzmann equation to the 2D incompressible Euler equations when the vorticity belongs to L^p.

Jiayun Meng, University of Texas at Austin

a-contraction theory for general viscous systems of conservation laws

The a-contraction theory is a recent energy-based method developed to tackle the stability and asymptotic limits for general solutions to conservation laws. In this talk, I will focus on the time asymptotic stability of composite waves of viscous shock and rarefaction for viscous conservation laws. I will describe how the a-contraction theory, first developed in this context to the compressible Navier-Stokes equation, can be extended to such general situations. This is a joint work with Young-Sam Kwon and Alexis Vasseur.

Matt Novack, Purdue University

Intermittent Weak Solutions of the 3D Euler Equations

In this talk, I will present a series of recent works, in part joint with H. Kwon, V. Giri, and V. Vicol. The common theme throughout is that the weak solutions we construct are intermittent; that is, they display deviations from the scaling laws predicted by Kolmogorov's 1941 theory of turbulence. The techniques we have developed allow us to (1) prove a "strong" version of Onsager's famous conjecture, and (2) construct solutions to 3D Euler with well-defined helicity which is not conserved.

Laurel Ohm, University of Wisconsin-Madison

Free boundary dynamics of an elastic filament in 3D Stokes flow

Motivated by biophysical applications, we consider a free boundary problem for a thin elastic filament immersed in 3D Stokes flow. The 3D fluid is coupled to the quasi-1D filament dynamics via a novel type of angle-averaged Neumann-to-Dirichlet operator. Much of the difficulty in the analysis lies in understanding this operator. We show that the principal part of this NtD map is the corresponding operator about a straight, periodic filament, for which we derive an explicit symbol. It is then possible to establish local well-posedness for an immersed filament evolving via a simple elasticity law. This establishes a mathematical foundation for the myriad computational results based on slender body approximations for thin immersed elastic structures.

Hyangdong Park, Korea Institute for Advanced Study(KIAS)

Shocks and contact discontinuities for flows with nonzero vorticity

We will discuss existence and stability of shocks and contact discontinuities for the steady Euler system. The Helmholtz decomposition method and the iteration method for 2-D and 3-D axisymmetric flows with nonzero vorticity will be introduced.

Steve Shkoller, University of California, Davis

Spacetime geometry of shock formation for the Euler equations in multiple space dimensions

While the compressible Euler equations are locally well-posed in Sobolev spaces with sufficient regularity, it is well-known that compressive initial conditions lead to a finite-time singularity at which certain components of the solution gradient become infinite.

While the Sobolev solution ceases to exist at this first singularity, at nearby spatial points the solution remains smooth. Our objective is to evolve the Euler solution past the first singularity, to determine the geometry of the spacetime set on which such a smooth solution can be continued, and identify the geometric coordinates and independent variables which yield uniform Sobolev bounds as the Eulerian gradient passes through a continuum of gradient catastrophes.

In this talk, we will describe a new Arbitrary Lagrangian Eulerian geometry adapted to the fast acoustic characteristic surfaces along which sound waves are propagated, and we will introduce a new set of Differentiated Riemann Variables that together allow for uniform Sobolev bounds of the Euler solutions with no derivative loss.

We will explain the geometry of the spacetime that contains such a smooth Euler solution: its future temporal boundary consists of the downstream singular set (the hypersurface of gradient catastrophes) and the upstream Cauchy horizon, which intersect on a co-dimension-2 set of pre-shocks.

This is joint work with Vlad Vicol at NYU.

Luis Silvestre, University of Chicago

The Landau equation does not blow up

The Landau equation is one of the main equations in kinetic theory. It models the evolution of the density of particles when they are assumed to repel each other by Coulomb potentials. It is a limit case of the Boltzmann equation with very soft potentials. In the space-homogeneous case, we show that the Fisher information is monotone decreasing in time. As a consequence, we deduce that for any initial data the solutions stay smooth and never blow up, closing a well-known open problem in the area.

Jakub Skrzeczkowski, University of Oxford

The Stein-log-Sobolev inequality and the exponential rate of convergence for the continuous Stein variational gradient descent method

The Stein Variational Gradient Descent method is a variational inference method in statistics that has recently received a lot of attention. The method provides a deterministic approximation of the target distribution, by introducing a nonlocal interaction with a kernel. Despite the significant interest, the exponential rate of convergence for the continuous method has remained an open problem, due to the difficulty of establishing the related so-called Stein-log-Sobolev inequality. Here, we prove that the inequality is satisfied for each space dimension and every kernel whose Fourier transform has a quadratic decay at infinity and is locally bounded away from zero and infinity. Moreover, we construct weak solutions to the related PDE satisfying exponential rate of decay towards the equilibrium. The main novelty in our approach is to interpret the Stein-Fisher information as a duality pairing between H^{-1} and H^1, which allows us to employ the Fourier transform. We also provide several examples of kernels for which the Stein-log-Sobolev inequality fails, partially showing the necessity of our assumptions. This is a joint work with J. A. Carrillo and J. Warnett.

Ian Tice, Carnegie Mellon University

Stationary and slowly traveling solutions to the free boundary Navier-Stokes equations

The stationary problem for the free boundary incompressible Navier-Stokes equations lies at the confluence of two distinct lines of inquiry in fluid mechanics. The first views the dynamic problem as an initial value problem. In this context, the stationary problem arises naturally as a special type of global-in-time solution with stationary sources of force and stress. One then expects solutions to the stationary problem to play an essential role in the study of long-time asymptotics or attractors for the dynamic problem. The second line of inquiry, which dates back essentially to the beginning of mathematical fluid mechanics, concerns the search for traveling wave solutions. In this context, a huge literature exists for the corresponding inviscid problem, but progress on the viscous problem was initiated much more recently in the work of the speaker and co-authors. For technical reasons, these results were only able to produce traveling solutions with nontrivial wave speed. In this talk we will discuss the well-posedness theory for the stationary problem and show that the solutions thus obtained lie along a one-parameter family of slowly traveling wave solutions. This is joint work with Noah Stevenson.

Hung Tran, University of Wisconsin-Madison

Recent progress on the study of a critical Coagulation-Fragmentation equation

I will discuss some recent results and ideas on the study of a continuous/discrete critical Coagulation-Fragmentation equation. I will highlight both the continuous and the discrete settings, old and new approaches.

Kristoffer Varholm, University of Pittsburgh

Vortex-carrying solitary gravity waves of large amplitude

In this paper, we study two-dimensional traveling waves in finite-depth water that are acted upon solely by gravity. We prove that, for any supercritical Froude number (non-dimensionalized wave speed), there exists a continuous one-parameter family C of solitary waves in equilibrium with a submerged point vortex. This family bifurcates from an irrotational uniform flow, and, at least for large Froude numbers, extends up to the development of a surface singularity. These are the first rigorously constructed gravity wave-borne point vortices without surface tension, and notably our formulation allows the free surface to be overhanging. We also provide a numerical bifurcation study of traveling periodic gravity waves with submerged point vortices, which strongly suggests that some of these waves indeed overturn. Finally, we prove that at generic solutions on C – including those that are large amplitude or even overhanging – the point vortex can be desingularized to obtain solitary waves with a submerged hollow vortex. Physically, these can be thought of as traveling waves carrying spinning bubbles of air.

Alexis Vasseur, University of Texas at Austin

Non-uniqueness for continuous solutions to 1D conservation laws

Abstract: In this talk, we show that a geometrical condition on 2×2 systems of conservation laws leads to non-uniqueness in the class of 1D continuous functions. This demonstrates that the Liu Entropy Condition alone is insufficient to guarantee uniqueness, even within the mono-dimensional setting. We provide examples of systems where this pathology holds, even if they verify stability and uniqueness for small BV solutions. Our proof is based on the convex integration process. Notably, this result represents the first application of convex integration to construct non-unique continuous solutions in one dimension. This is a joint work with Robin Ming Chen, and Cheng Yu.

Dehua Wang, University of Pittsburgh.

Gauss-Codazzi equations and isometric immersions

The isometric embedding of surfaces in geometry can be formulated as an initial/boundary value problem for the Gauss-Codazzi equations. We shall review the related results on global solutions, and discuss a recent result on the global smooth solutions when the surface has a finite total curvature.

Weiqiang Wang, University of Pittsburgh

Low Mach number Limit of Steady Thermally Driven Fluid

We consider the existence of strong solutions to the steady non-isentropic compressible Navier-Stokes system with Dirichlet boundary conditions in bounded domains where the fluid is driven by the wall temperature, and study its low Mach number limit, i.e., $\varepsilon \to 0$.

Based on a new expansion with respect to ε and an elegant $\varepsilon-$dependent higher order energy estimates, we establish the existence of the strong solutions and justify its low Mach number limit in L^∞ sense with a rate of convergence. Notably, for the limiting system obtained in the low Mach number limit, the variation of the wall temperature is allowed to be independent of the Mach number. It is also worth pointing out that the velocity field u_1 acts like a ghost since it appears at ε-order in the expansion, but still affects the density and temperature at $O(1)$-order. This is a joint work with Professors Feimin Huang and Yong Wang.